

New Minimal SO(10) GUT : A Theory for All Epochs*

Charanjit S. Aulakh^{†1,2}

¹*Dept. of Physics, Panjab University*

Chandigarh, 160014, India

²*Indian Institute of Science Education and Research Mohali,*

Sector 81, S. A. S. Nagar, Manauli PO 140306, India

Abstract

The Supersymmetric SO(10) theory (“NMSO(10)GUT”) based on the $\mathbf{210} + \mathbf{126} + \overline{\mathbf{126}}$ Higgs system proposed in 1982 has evolved into a realistic theory capable of fitting the known low energy Particle Physics data besides providing a Dark matter candidate and embedding Inflationary Cosmology. It dynamically resolves longstanding issues such as fast dimension five operator mediated proton decay in Susy GUTs by allowing explicit and complete calculation of crucial threshold effects at M_{Susy} and M_{GUT} in terms of fundamental parameters. This shows that SO(10) Yukawas responsible for observed fermion masses as well as operator dimension 5 mediated proton decay can be highly suppressed on a “Higgs dissolution edge” in the parameter space of GUTs with rich superheavy spectra. This novel and generically relevant result highlights the need for every realistic UV completion model with a large/infinite number of heavy fields coupled to the light Higgs doublets to explicitly account for the large wave function renormalization effects on emergent light Higgs fields in order to be considered a quantitatively well defined candidate UV completion. The NMSGUT predicts large soft Susy breaking trilinear couplings and distinctive sparticle spectra. Measurable or near measurable level of tensor perturbations- and thus large Inflaton mass scale- may be accommodated by Supersymmetric Seesaw inflation within the NMSGUT based on an LHN flat direction Inflaton if the Higgs component contains contributions from heavy Higgs components. Successful NMSGUT fits suggest a *renormalizable* Yukawon Ultra minimal gauged theory of flavor based upon the NMSGUT Higgs structure.

* To appear in the proceedings of UNICOS2014 [1]

[†] aulakh@iisermohali.ac.in; 14.139.227.202/Faculty/aulakh/

I. INTRODUCTION

Grand Unification theories (GUTs) have seen some 40 summers since the basic idea was first proposed by Pati and Salam in 1974[2] yet retain their attraction as the most obvious progression of the gauge logic embodied in the standard model. Indeed, within Particle physics, the only tangible and clear hints of Physics beyond the Standard model are the remarkable convergence of gauge couplings in the Unification regime : $10^{15} - 10^{17}$ GeV and the existence of neutrino masses (initially thought to be superfluous in both the Electro-weak and GUT contexts). To this one should perhaps append the convergence of the pivotal[3] third generation Yukawa couplings in the MSSM at high $\tan \beta$. The same gauge logic that structures the SM (and justifies the zeroth order decoupling of right handed neutrinos) also makes the Type I seesaw mechanism[4] the most natural rationale for the milli-eV range neutrino masses actually observed. Supersymmetry seems essential to the structural stability of the SM and experience[5, 6] with supersymmetric LR models and renormalizable SO(10) GUTs shows it ensures complete calculability for UV completions of the MSSM. Thus non-discovery of sparticles in the searches so far has still not dimmed the ardor of its many adherents. Arguments for considering some variety of the MSSM as the effective theory of any GUT became all the more cogent when, with the observation of solar and atmospheric neutrino oscillations at Super-Kamiokande, the scale of B-L breaking associated with a renormalizable seesaw mechanism emerged ($V_{EW}^2/(10^{-12}GeV) \sim 10^{14}$ GeV) in the GUT ball-park. The convergence of these two compelling arguments thus hints at a deep interconnection between B and L violation in a Supersymmetric Grand Unified framework[7, 8]. It is SO(10) *not* SU(5) GUTs that provide the most natural GUT framework, free of gauge singlets (which are really opaque, and thus anathemic, to the gauge logic pursued so successfully in the SM), for Type I (and also Type II) seesaw. Thus eventually SO(10) GUTs displaced SU(5) theories and achieved a long delayed vindication and recognition as minimal UV complete frameworks for neutrino oscillations. It was only natural that our pursuit of consistent minimal Left right supersymmetric R-parity preserving models[5] from the mid-nineties led in short order[6] after the epochal measurements of Super-Kamiokande to a realization that the very first complete supersymmetric SO(10) GUT[9, 10] proposed way back in 1982, just after the Georgi-Dimopoulos minimal Susy SU(5) model[11], was in fact the Minimal Supersymmetric GUT(MSGUT)[12]. This model is now called[13]- due

to a transient glory of a truncated version with only $\mathbf{10} + \overline{\mathbf{126}}$ *but not* $\mathbf{120}$ coupling to matter fermion bilinears as the Minimal theory[14] - the New/Next MSGUT(NMSGUT). In the best Popperian mode it has survived another decade of detailed investigation of its ability to fit all available SM, gauge, fermion and neutrino data, as well as the consistency of one-loop threshold corrections at both low and high energy scales made possible by its extreme parameter economy(a.k.a. minimality) and full calculability : simple virtues that are, alas, all too rare among the plethora of its (non-minimal) competitors. The model is based on a $\mathbf{210} \oplus \mathbf{126} \oplus \overline{\mathbf{126}}$ GUT Higgs system and already in 1982[9] showed clearly that prevention of a RG flow catastrophe due to large pseudo-goldstone supermultiplets requires a single step breaking of Susy SO(10) to the MSSM : later rediscovered in the context of another R-parity preserving but non-minimal model[6]. Being a complete theory of gauge physics the model is capable of fitting and/or predicting most, if not all, of the commonly considered varieties of BSM physics, including neutrino masses, the g-2 muon anomaly, LSP dark matter, B violation (at acceptable rates : see below), Leptogenesis driven by the parameters controlling neutrino mass etc. Remarkably it also comes close to providing a workable embedding of inflation based upon the D-flat directions involving Lepton and Higgs doublet fields naturally present and contributing to the effective MSSM below the GUT scale. The model also predicts a distinctive and characteristic *normal* hierarchy of sfermion generations discoverable by the LHC or its successors. As striking was its early indication, on the basis of the parameters required for a successful fermion fit, that the trilinear soft susy breaking parameter A_0 *must* be large (as far back as 2008[15] i.e well before Higgs mass discovery made such a large A_0 respectable rather than ludicrously fine tuned: as then argued by naturalists).

The fit the of low energy data in terms of fundamental parameters obtained in the NMSGUT must further be consistent with, indeed predict, a vast range of phenomena ranging from exotic processes such as Baryon number and Lepton flavour violation to cosmological Leptogenesis, Dark Matter relic density and inflation. The demonstration of the feasibility of such comprehensive fits of the SM data directly in terms of Susy GUT parameters marks the NMSGUT as being cast in the mould of the SM rather than its piece meal generalizations. The most striking themes and results that have emerged recently from our detailed investigation of the structural freedoms-in-necessity granted by the full calculability NMSGUT are:

1. A simple generic mechanism for suppression of the long problematic[16] $d = 5$ super fast proton decay generic to Susy GUTs.
2. An embedding of high scale supersymmetric renormalizable inflexion[17] based upon the left Lepton(L) -Higgs doublet(H) - conjugate Neutrino($N \equiv \nu_L^c$) flat direction (labelled by the LHN chiral invariant) and utilizing the involvement of heavy partners of the MSSM Higgs. This may prove very attractive in the context of the BICEP2 driven focus on high scale inflation.
3. Completely novel “Grand Yukawonification” models based upon gauging of an $O(3)$ subgroup of the $U(3)$ flavour symmetry of the $SO(10)$ GUT fermion kinetic terms. This flavour symmetry is naturally broken at the GUT scale by the NMSGUT Higgs fields which promote themselves rather naturally to also carry the function of being Yukawons i.e. fields whose vevs determine the Yukawa couplings in the effective MSSM besides simultaneously breaking $SO(10)$ *while maintaining renormalizability*.
4. The models in item 3. above are made possible by a novel conflation of supergravity mediated supersymmetry breaking with the breaking of gauged flavour symmetry in the Hidden sector of the model using the so called Bajc-Melfo calculable metastable Susy breaking vacua[18, 20].

Thus the NMSGUT potentially provides a completely realistic and predictive theory of particle physics in all energy ranges and cosmological epochs. We have even speculated[21] that the Landau pole in the NMSGUT gauge coupling which occurs quite near the Planck scale should be interpreted as a physical cutoff on the perturbative dynamics associated with the scale where the NMSGUT condenses(via a strong coupled supersymmetric dynamics) in the ultra violet. This scale could then function as the Planck scale of an effective induced supergravity at large length scales emergent from the NMSGUT when the metric and gravitino fields introduced to define a coordinate independent microscopic GUT acquire kinetic terms due to quantum effects. Since items 2) 3) are published as [17, 18] and are also reported in these proceedings in the contributions of my collaborators Ila Garg and Charanjit Kaur respectively, I will touch upon them only briefly but focus on the results in 1), 4) and discuss some of their implications while referring the reader to the published papers[12, 13, 22, 23, 25, 26] for details.

II. NMSGUT BASICS

The NMSGUT superpotential is built from the quadratic SO(10) invariants and associated mass parameters

$$m : \mathbf{210}^2 \quad ; \quad M : \mathbf{126} \cdot \overline{\mathbf{126}} \quad ; \quad M_H : \mathbf{10}^2 \quad ; \quad m_\Theta : \mathbf{120}^2 \quad (1)$$

and trilinear couplings :

$$\begin{aligned} \lambda : \mathbf{210}^3 \quad ; \quad \eta : \mathbf{210} \cdot \mathbf{126} \cdot \overline{\mathbf{126}} \quad ; \quad \rho : \mathbf{120} \cdot \mathbf{120} \cdot \mathbf{210} \\ k : \mathbf{10} \cdot \mathbf{120} \cdot \mathbf{210} \quad ; \quad \gamma \oplus \bar{\gamma} : \mathbf{10} \cdot \mathbf{210} \cdot (\mathbf{126} \oplus \overline{\mathbf{126}}) \\ \zeta \oplus \bar{\zeta} : \mathbf{120} \cdot \mathbf{210} \cdot (\mathbf{126} \oplus \overline{\mathbf{126}}) \\ \mathbf{16_A} \cdot \mathbf{16_B} \cdot (h_{AB} \mathbf{10} + f_{AB} \overline{\mathbf{126}} + g_{AB} \mathbf{120}) \end{aligned} \quad (2)$$

The couplings $h, f(g)$ are complex (anti)-symmetric in the flavour indices due to properties of the SO(10) Clifford algebra. Either h or f is chosen real and diagonal using the $U(3)$ flavor symmetry of the matter kinetic terms. The matter Yukawas thus contain 21 real parameters. Five phases say of $m, M, \lambda, \gamma, \bar{\gamma}$ are set to zero by phase conventions. One (complex parameter) say M_H , is fine tuned to keep two Higgs doublets (H, \bar{H}) of the effective MSSM light, leaving 23 magnitudes and 15 phases as parameters. Fine tuning fixes H, \bar{H} composition as a mixture of the (6 pairs of the MSSM type) doublet fields in the GUT as a function of the superpotential parameters entering the null vectors of the $G_{3,2,1}$ irreps $[1, 2, \pm 1]$ mass terms. The mixture is described by the so called ‘‘Higgs fractions’’ [12, 23].

The GUT scale vevs(units m/λ) are known functions of x which is a solution of the cubic ($\xi = \frac{\lambda M}{\eta m}$)

$$8x^3 - 15x^2 + 14x - 3 + \xi(1 - x)^2 = 0 \quad (3)$$

The complete set of GUT scale mass matrices for the 26 different MSSM irrep types and tree level low energy effective Superpotential was calculated [13] extending the result for the MSGUT[22, 23, 26, 27]. Using these and two loop RG flows the gauge, superpotential and soft susy breaking parameters at GUT scales can be matched, using a downhill simplex search procedure, to the known values of the MSSM data described above[13]. This also yields a mini-split (10-100 TeV) supersymmetry particle spectrum with large A_0, μ parameters, light gauginos and Bino LSP, super heavy Higgsinos, sfermions in tens of TeV and sometimes

a smuon(or other sfermion) light enough (i.e within 10% of the light Bino LSP) to co-annihilate with it and provide acceptable dark matter relic density[29]. The light smuon case is obviously attractive if Supersymmetry is called upon to explain the muon magnetic moment anomaly and emerges in the right ball park for these solutions. Flavour violation in the Quark and Lepton sectors is also well controlled because of the multi-TeV masses of most of the sfermions. Moreover A_0 is required to be large to allow the b quark mass to be fitted and this was found[13, 15] serendipitously *before* Higgs discovery made it a requirement for Supersymmetry.

III. GUT SCALE THRESHOLD CORRECTIONS AND BARYON DECAY RATE

The SO(10) parameters determined by the realistic fit are substituted into the effective (dimension four) superpotential describing Baryon violation which, after RG flow to low energies, is used to calculate the proton decay rate. As is well known the generic result gives a lifetime of some 10^{27} years i.e some 7 orders of magnitude shorter than the current limits from the Super Kamiokande experiment. The beautifully complete and predictive fits are thus of little use if this problem remains unsurmountable. However, as in previous tight spots, the NMSGUT points out a convincing and illuminating, generically applicable and dynamical, pathway out of the difficulty : precisely because of the available explicit solution described above.

Superpotential parameters renormalize only by wave function corrections. In the computable basis sets where heavy supermultiplet masses are diagonal a generic heavy field type Φ (conjugate $\bar{\Phi}$) mass matrix diagonalizes as :

$$\bar{\Phi} = U^\Phi \bar{\Phi}' \quad ; \quad \Phi = V^\Phi \Phi' \quad \Rightarrow \quad \bar{\Phi}'^T M \Phi = \bar{\Phi}'^T M_{Diag} \Phi' \quad (4)$$

Circulation of heavy within light field propagators entering the matter(f_A, f_A^c)-Higgs Yukawa vertices : $\mathcal{L} = [f_c^T Y_f f H_f]_F + H.c. +$ implies [30] a finite wave function renormalization in the fermion and Higgs Kinetic terms

$$\mathcal{L} = \left[\sum_{A,B} (\bar{f}_A^\dagger (Z_{\bar{f}})_A^B \bar{f}_B + f_A^\dagger (Z_f)_A^B f_B) + H^\dagger Z_H H + \bar{H}^\dagger Z_{\bar{H}} \bar{H} \right]_D + .. \quad (5)$$

Unitary matrices $U_{Z_f}, \bar{U}_{Z_{\bar{f}}}$ diagonalize $(U^\dagger Z U = \Lambda_Z) Z_{f,\bar{f}}$ to positive definite form $\Lambda_{Z_f, Z_{\bar{f}}}$. We define a new basis to put the Kinetic terms of the light matter and Higgs fields in

canonical form :

$$\begin{aligned} f &= U_{Z_f} \Lambda_{Z_f}^{-\frac{1}{2}} \tilde{f} = \tilde{U}_{Z_f} \tilde{f} \quad ; \quad \bar{f} = U_{Z_{\bar{f}}} \Lambda_{Z_{\bar{f}}}^{-\frac{1}{2}} \tilde{\bar{f}} = \tilde{U}_{Z_{\bar{f}}} \tilde{\bar{f}} \\ H &= \frac{\tilde{H}}{\sqrt{Z_H}} \quad ; \quad \bar{H} = \frac{\tilde{\bar{H}}}{\sqrt{Z_{\bar{H}}}} \end{aligned} \quad (6)$$

Thus when matching to the effective MSSM it is Yukawa couplings of the effective MSSM \tilde{Y}_f :

$$\tilde{Y}_f = \Lambda_{Z_f}^{-\frac{1}{2}} U_{Z_f}^T \frac{Y_f}{\sqrt{Z_{H_f}}} U_{Z_f} \Lambda_{Z_f}^{-\frac{1}{2}} = \tilde{U}_{Z_f}^T \frac{Y_f}{\sqrt{Z_{H_f}}} \tilde{U}_{Z_f} \quad (7)$$

and not the original tree level ones that match the MSSM at the matching scale.

Light Chiral field Φ_i the corrections have generic form ($Z = 1 - \mathcal{K}$) :

$$\mathcal{K}_i^j = -\frac{g_{10}^2}{8\pi^2} \sum_{\alpha} Q_{ik}^{\alpha} Q_{kj}^{\alpha} F(m_{\alpha}, m_k) + \frac{1}{32\pi^2} \sum_{kl} Y_{ikl} Y_{jkl}^* F(m_k, m_l) \quad (8)$$

where $\mathcal{L} = g_{10} Q_{ik}^{\alpha} \psi_i^{\dagger} \gamma^{\mu} A_{\mu}^{\alpha} \psi_k$ describes the generic gauge coupling and $W = \frac{1}{6} Y_{ijk} \Phi_i \Phi_j \Phi_k$ the generic Yukawa couplings, while $F(m_1, m_2)$ are symmetric 1-loop Passarino-Veltman functions.

Crucially the SO(10) Yukawa couplings $(h, f, g)_{AB}$ also enter into the coefficients L_{ABCD}, R_{ABCD} of the $d = 5$ baryon decay operators in the effective superpotential obtained by integrating out the heavy chiral supermultiplets that mediate baryon decay[13, 22, 23].

\tilde{Y}_f must be diagonalized to mass basis (denoted by primes) so that $d = 5, \Delta B = \pm 1$ decay operator coefficients become

$$\begin{aligned} L'_{ABCD} &= \sum_{a,b,c,d} L_{abcd} (\tilde{U}'_Q)_{aA} (\tilde{U}'_Q)_{bB} (\tilde{U}'_Q)_{cC} (\tilde{U}'_L)_{dD} \\ R'_{ABCD} &= \sum_{a,b,c,d} R_{abcd} (\tilde{U}'_{\bar{e}})_{aA} (\tilde{U}'_{\bar{u}})_{bB} (\tilde{U}'_{\bar{u}})_{cC} (\tilde{U}'_{\bar{d}})_{dD} \end{aligned} \quad (9)$$

The search for a fit with the constraint that L'_{ABCD}, R'_{ABCD} be sufficiently suppressed (i.e yielding proton lifetime $\tau_p > 10^{34}$ yrs) then flows invariably towards parameter regions where $Z_{H, \bar{H}} \ll 1$ so SO(10) Yukawa couplings required to match the MSSM are greatly reduced. The same couplings enter L'_{ABCD}, R'_{ABCD} quadratically thus suppressing them drastically. This mechanism is generically available to realistic multi-Higgs theories. Indeed we go so far as to say that *any* UV completion incapable of performing this computation will remain less than quantitative and thus is no viable completion proposal at all. A tedious calculation determines the threshold corrections, see [31, 32] for details. $Z \simeq 0$ also leads

Solution	$\tau_p(M^+\nu)$	$\Gamma(p \rightarrow \pi^+\nu)$	$\text{BR}(p \rightarrow \pi^+\nu_{e,\mu,\tau})$	$\Gamma(p \rightarrow K^+\nu)$	$\text{BR}(p \rightarrow K^+\nu_{e,\mu,\tau})$
1	9.63×10^{34}	4.32×10^{-37}	$\{1.3 \times 10^{-3}, 0.34, 0.66\}$	9.95×10^{-36}	$\{4.6 \times 10^{-4}, 0.15, 0.85\}$
2	3.52×10^{34}	2.14×10^{-36}	$\{1.7 \times 10^{-3}, 0.18, 0.81\}$	2.62×10^{-35}	$\{1.8 \times 10^{-3}, 0.19, 0.81\}$

TABLE I: $d = 5$ operator mediated proton lifetimes $\tau_p(\text{yrs})$, decay rates $\Gamma(\text{yr}^{-1})$ and Branching ratios in the dominant Meson $^+ + \nu$ channels.

to smaller GUT couplings and compressed heavy spectra compared to previous fits. Higher loop corrections seem computationally prohibitive. However we have calculated[28, 29, 37] the complete SO(10) two loop beta functions and two loop threshold corrections also rely upon the same anomalous dimensions. So convoluting GUT scale mass spectra with our SO(10) loop sums determines two loop threshold corrections as well. Recovering our one loop results by this method is the necessary first step to proceed in this direction[38].

Searches for fits using the threshold corrected Baryon Decay operators yield s-spectra similar to those found earlier, but with smaller values of all couplings (because of the constraints $Z_{H,\bar{H}} > 0$ imposed on the searches: which are badly violated if almost any of the super potential parameters grow large), and acceptable $d = 5$ B decay rates: as shown in Table 1. Imposing $\text{Max}|O^{(4)}| < 10^{-22}\text{GeV}^{-1}$ gives proton lifetimes above 10^{34} yrs. $Z_{H,\bar{H}}$ approach zero (from above) while $Z_{f,\bar{f}}$ are close to 1 : since **16**-plet Yukawas are all suppressed. See [28, 29, 31, 32] for further details of these and many related issues.

Our fits are associated with very distinctive sparticle spectra. An example is shown in Table 2 which is of the general type used to evaluate the B-violation rates seen in Table 1. Note the peculiar and remarkable smallness of the smuon mass which is a consequence of an RG flow driven by special features of the two loop RG flow of soft susy breaking parameters in models with $M_H^2 > M_{\bar{H}}^2 < 0$. As discussed in [31] the RG flow in such theories can be such as to drive $M_{\tau_R}^2$ first negative and then to large positive values(due to the large value of the third generation Yukawa coupling) while in the relatively flat evolution of the first two s-generations the R-smuon lags the R-stau and the R-selectron the R-smuon due to the significant difference in their Yukawa couplings. This can result in the peculiar feature that the running mass squared of the smuon is smallest of all the sfermion masses and close to the LSP mass. Then the smuon(or other light sfermion since in other cases up or down squarks of the light generations may emerge lightest) provides the necessary co-annihilation

channel for achieving an acceptable Bino LSP dark matter [29]. The smuon case is doubly attractive since it will also contribute to the muon $g-2$ anomaly. We note however that the sfermion spectra shown here are not yet loop corrected and that the loop corrected spectra in solutions where we have found so far do not present this feature.

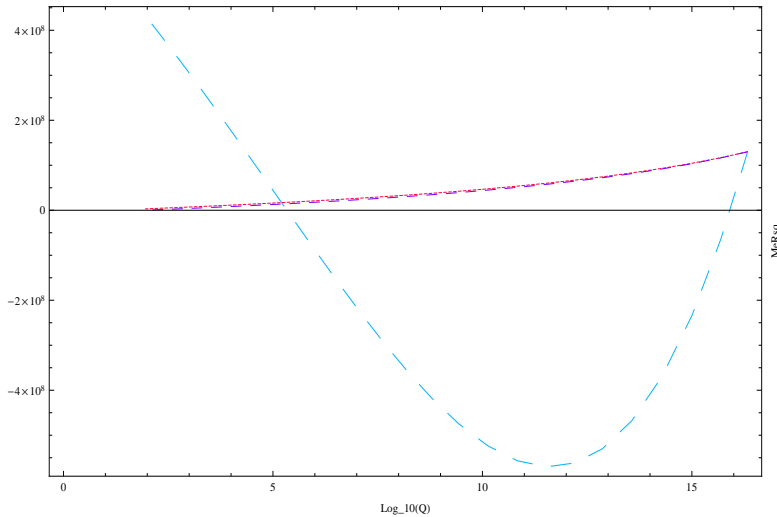


FIG. 1: Running right-slepton masses squared showing development of small R-smuon(blue-medium dashed) as it leads the R-selectron(small dash-red) and far lags the R-stau(large blue dashes) which first turns negative and then becomes large driven by large third generation Yukawa coupling

Besides the immediate satisfaction of finding our model may be realistic and viable, our calculations underline that the nature of the competition among candidate UV completions has been modified by our results. Candidate (Susy) UV completion models with a large or infinite number of fields must not only show that their parameters can consistently be constrained to yield some variant of the (MS)SM *but also* that modification of light wave functions by coupling to heavy fields is both *calculable* and sensible in the sense of the possibilities considered and demonstrated above. Absent such a calculability and consistency, a UV completion will remain at the level of a phantasmal possibility rather than a scientific model capable of braving this falsification gauntlet to progress to the status of a scientific Theory. We have thus shown that for the NMSGUT careful attention to the quantum communication between the low energy effective theory and the UV completion *through the light Higgs Portal* yields natural and generic suppression of fast proton decay in

Field	Mass(GeV)
$M_{\tilde{G}}$	1000.14
M_{χ^\pm}	569.81, 125591.22
M_{χ^0}	210.10 _{LSP} , 569.81, 125591.20, 125591.20
$M_{\tilde{\nu}}$	15308.069, 15258.322, 21320.059
$M_{\tilde{e}}$	1761.89, 15308.29, 211.57_{smuon} , 15258.60, 20674.72, 21419.56
$M_{\tilde{u}}$	11271.80, 14446.76, 11270.63, 14445.80, 24607.51, 40275.87
$M_{\tilde{d}}$	8402.99, 11272.10, 8401.48, 11270.95, 40269.19, 51845.93
M_A	377025.29
M_{H^\pm}	377025.30
M_{H^0}	377025.28
M_{h^0}	124.00_{h^0}

TABLE II: Soft Susy breaking spectrum: Large μ, B, A_0 Bino LSP. Light gauginos, Normal S-hierarchy Higgs (h^0) as found , Light smuon ! Other sfermions multi-TeV : Decoupled & Mini-split, large μ, A_0

Susy GUTs[32] due to the cumulative effects of the large number of GUT fields that allow the theory to approach and live on the “Higgs dissolution edge” by renormalizing the light Higgs wavefunction and thus its Yukawa couplings, even when individual GUT couplings are all well within the perturbative regime.

IV. SUPERSYMMETRIC SEESAW INFLATION

The necessity of cosmological Inflation and even its description in terms of a single ‘Inflaton’ slowly rolling down a potential plateau before oscillating around its true minimum to produce quanta of the low energy theory in the post-inflationary reheating regime is by now well accepted[39]. However the provenance of the Inflaton field and potential is faced by an embarrassing multiplicity of candidates: many of which are compatible[39] with even the latest data[40]. From a Particle Physics- rather than gravitation physics- viewpoint identification of an Inflaton candidate from among the known particle degrees of freedom is most appealing. The suggestion [41] to utilize one (or more) of the D-flat directions(labelled

by the Chiral gauge invariants) of the MSSM Lagrangian as an inflaton candidate has thus enjoyed a vogue. Such a suggestion does however face various stringent constraints such as ensuring a suitable scale of inflation together with the tiny Yukawa couplings for matter fermions required to ensure flatness of the potential during inflation. Since neutrino-lepton Yukawa couplings may well be small flat directions with sneutrino and lepton components are well suited for this role. In [42] an MSSM extended by conjugate neutrinos($N \equiv \nu_L^c$) superfields to allow (tiny) *Dirac* neutrino masses and an extra gauge generator(B-L) was used as a framework for an inflaton made up of the Lepton and Higgs doublets (the flat direction is labelled as $\phi \equiv LHN$). Provided a stringent fine tuning is made between the trilinear soft susy breaking A_ϕ and the inflaton mass ϕ enjoys a renormalizable potential characterized by an inflection point at $\phi_0 \sim m_\phi/h_\nu$ with $m_\phi \sim 100 \text{ GeV} - 10 \text{ TeV}$, $h_\nu \sim 10^{-12}$. Then $\phi_0 \sim 10^{11} - 10^{15} \text{ GeV}$ and the energy density is characterized by a scale $V_0^{1/4} \equiv \Lambda \sim m_\phi/\sqrt{h_\nu} \sim 10^8 - 10^{10} \text{ GeV}$. This model is extremely fine tuned[19] since the inflaton mass M_ϕ is set by the Susy breaking scale the TeV range. Moreover since susy breaking parameters are subject to radiative corrections it seems strange to impose extreme fine tuning on the trilinear coupling of the inflaton which arises only from the soft supersymmetry breaking parameters. It was natural[17] for us to ask the consequences of Seesaw[4] masses for the light neutrinos i.e. heavy Majorana masses for the right handed neutrinos and *consequently* tiny Majorana masses for the left handed neutrinos. In this case we showed that the soft parameters were irrelevant, tuning was done in the superpotential, and the large right handed neutrino masses($m_\phi \sim M_{\nu^c} \sim 10^6 - 10^{12} \text{ GeV}$) and reasonable superpotential yukawas $h_\nu \sim 10^{-13}(M/\text{GeV})^{1/2}$ which could thus be as large as 10^{-6} and have mild fine tuning: rather than the radiatively unstable and highly fine tuned[19] soft symmetry breaking parameters of the Dirac case. The embedding of the Supersymmetric Seesaw Inflation(SSi) model in the NMSGUT was also attempted but could not achieve a large enough number of e-folds. However the advent of BICEP2 has reopened the whole question of the inflaton mass and suddenly made it plausible that it might be anywhere in the range between $M_{\nu^c} - M_{GUT}$! The ratio of tensor to scalar modes r emerges as $r \sim 2(m_\phi/(10^{14} \text{ GeV}))^3$ but r will be measurable in the near term only if $r > 10^{-1.5}$. If the value of r is in this ball park then the inflaton mass would need to be much larger than even the commonly accepted upper limit of around 10^{12} GeV on the right handed neutrino masses. Interestingly the NMSGUT embedding studied by us earlier allows a generalization

where mass of superheavy Higgs doublets mixed into the inflaton control its mass allowing inflaton masses right upto the BICEP2 indicated range. This scenario allows raising the achievable number of e-folds by around 5 orders of magnitude ! The contribution of Ila Garg in this volume provides further details.

V. GRAND YUKAWONIFICATION AND THE FLAVOR-SUSY BREAKING LINK

The NMSO(10)GUT has achieved gauge and third generation yukawa unification consistent with B violation limits, along with excellent fits of other known fermion data. It makes and intriguing and distinctive predictions regarding sparticle spectra, and may well be compatible with the dominant Cold Dark Matter plus Inflation plus Leptogenesis cosmological scenario specially if the scale of Inflation turns out to be near that suggested by BICEP2[43]. It is only natural to ask whether so complete a model can pretend to shed any light on the most outstanding mystery of particle physics : the flavor puzzle a.k.a the fermion hierarchy i.e. the origin of the peculiar large inter-generational ratios of charged fermion masses combined with small quark but two large lepton mixing angles. At first sight, since we impose no discrete symmetries and since yukawa couplings are parameters dialed to fit the data, it seems that this is asking too much. However reflection on the way in which the SM Yukawa couplings emerge and the fact that the *only* convincing hint of flavor unification seems to be the convergence of third generation Yukawa couplings at the GUT scale suggests that perhaps the Yukawa couplings may emerge via spontaneous breaking of the U(3) flavour symmetry of the SO(10) invariant matter kinetic terms. In [18] we suggested that our work[9, 12, 13, 23, 25, 32] on MSGUTs naturally and minimally identifies MSGUT Higgs multiplets as candidate Yukawon multiplets. It thus yields a novel mechanism whereby the fermion hierarchy could emerge from a flavour symmetric and renormalizable GUT . In our work “Yukawons” *also carry representations of the gauge (SM/GUT) dynamics*. In previous work typically the dimension 1 Yukawa-on \mathcal{Y} in the Higgs vertex makes it non-renormalizable ($\mathcal{L} = f^c \mathcal{Y} f H / \Lambda_{\mathcal{Y}} + \dots$: where the unknown high scale $\Lambda_{\mathcal{Y}}$ controls Yukawa-on dynamics) . Minimal SO(10) GUTs[9, 10, 12, 13] provide a gauged $O(N_g)$ family symmetry route to “Yukawonification” : with the GUT and family symmetry breaking at the same scale: obviating the need for non-renormalizable interactions and any

extraneous scale Λ_Y . As we shall see the consistency conditions for the maintainability of the Higgs Portal to the UV completion of the MSSM play a central role in determining just how the peculiarly lopsided and “senseless” fermion hierarchy is produced from the flavour symmetric and Grand Unified UV completion. In minimal Susy GUTs one eschews invocation of discrete symmetries and insists only upon following the logic of $SO(10)$ gauge symmetry. This insistence, combined with careful attention to the implications of the emergence of a single light MSSM Higgs pair from the $2N_g(N_g + 1)$ pairs in the $O(N_g)$ extended MSGUT, leads to an effectively unique extension of the $SO(10)$ gauge group by a $O(N_g)$ family gauge symmetry for the N_g generation case and the dynamical emergence of fermion hierarchy and mixing. The $\overline{\mathbf{126}}(\overline{\Sigma})$ also contributes to both neutrino and charged fermion masses. Gauging just an $O(N_g)$ subgroup of the $U(N_g)$ symmetry of the fermion kinetic terms seems the workable option : in contrast to a unitary family group, because the use of complex representations introduces anomalies and requires doubling of the Higgs structure to cancel anomalies and to permit holomorphic invariants to be formed for the superpotential. Worse, Unitary symmetry enforces vanishing of half the emergent matter Yukawa couplings. $O(N_g)$ family symmetry suffers from none of these defects and gauging it ensures that no Goldstone bosons arise when it is spontaneously broken. We emphasize that in contrast with previous ‘spurion/Yukawa-on’ (see e.g. [44]) our model is renormalizable and GUT based.

The GUT superpotential has *exactly* the same form as the MSGUT (See [13, 22, 23, 26] for comprehensive details) :

$$\begin{aligned}
W_{GUT} &= \text{Tr}(m\Phi^2 + \lambda\Phi^3 + M\overline{\Sigma}.\Sigma + \eta\Phi.\overline{\Sigma}.\Sigma) \\
&\quad + \Phi.H.(\gamma\Sigma + \bar{\gamma}.\overline{\Sigma}) + M_H H.H) \\
W_F &= \Psi_A.((hH) + (f\Sigma) + (g\Theta))_{AB}\Psi_B
\end{aligned} \tag{10}$$

We have shown how the **120**-plet is included in W_F but have studied only MSGUTs (i.e with **10**, $\overline{\mathbf{126}}$). Inclusion of the **120**-plet does not affect GUT SSB. The *only* innovation in Higgs structure is that all the MSGUT Higgs fields now carry symmetric representation of the $O(N_g)$ family symmetry : $\{\Phi, \overline{\Sigma}, \Sigma, H\}_{AB}$; $A, B = 1, 2..N_g$ (under which the matter **16**-plets ψ_A are vector N_g -plets). Couplings h, f, g are single complex numbers while the Yukawons carry symmetric (**H**, $\overline{\Sigma}$) and anti-symmetric (**Θ**) representations of $O(3)$: as required by the transposition property of relevant $SO(10)$ invariants. For $N_g = 3$, real fermion mass parameters come down from 15 ($Re[h_{AA}], f_{AB}$) to just 3 ($Re[h], f$) without the **120**-plet (6

additional to just 2 with the **120**-plet). Thus this type of renormalizable flavour unified GUTs can legitimately be called Yukawon Ultra-Minimal GUTs(**YUMGUTs**). Further details on the gauge symmetry breaking and determination of Yukawas can be found in the contribution of Charanjit Kaur in this volume. We will focus on our resolution[20] of a severe technical difficulty that crops up when we spontaneously break a gauged $O(3)$ subgroup of the $U(3)$ supersymmetric flavour symmetry. If this difficulty is resolved the determination of the viability of the “Grand Yukawonification” model then becomes a matter of searching the relatively small remaining parameter space for viable parameter sets that fit the fermion data at M_X while taking account of threshold corrections at low and high scales and while respecting constraints on crucial quantities like the proton lifetime[13, 23, 32]. Note that in this approach not only are the hard parameters of the visible sector superpotential reduced by replacement of the flavoured parameters by bland family symmetric ones but also the soft supersymmetry breaking parameters are determined by the two parameters of the hidden sector superpotential and the Planck scale.

With just the family index carrying MSGUT Higgs present the flavor D terms cannot vanish. So additional fields with vevs free to cancel the contribution \bar{D}_X^A of the GUT sector to $O(N_g)$ D terms are needed. The extra F terms must be sequestered from the GUT sector to preserve the MSGUT SSB. The special role of the Bajc-Melfo supersymmetry breaking is that it provides flat directions in both the singlet and the gauge variant parts of a symmetric chiral supermultiplet $S_{AB} = S_{BA}$. Since it is very difficult to make the contribution of the visible sector GUT fields to the $O(N_g)$ D-terms vanish, the \hat{S} flat direction performs the invaluable function of cancelling this contribution *without disturbing the symmetry breaking in the visible sector*. We proposed [20] Bajc-Melfo type two field superpotentials [33, 34] (of structure $W = S(\mu_B\phi + \lambda_B\phi^2)$). Their potentials have local minima breaking supersymmetry ($\langle F_S \rangle \neq 0$) which leave the vev $\langle S \rangle$ undetermined (ϕ gets a vev). If ϕ, S transform in symmetric representations of $O(N_g)$ then the undetermined vev of the trace free part of S is determined at the global supersymmetry level by the minimization of the $O(N_g)$ D-terms thus preserving Supersymmetry from high scale breaking. On the other hand, BM superpotentials and their metastable (local) supersymmetry breaking vacua function efficiently[20] as hidden sectors for supergravity GUT models : coupling to supergravity determines the $O(N_g)$ singlet part of S to have a vev of order the Planck scale. This felicitous and unexpected marriage of the NMSGUT to gauged flavor symmetry and Supergravity reduces the

number of $SO(10)$ yukawas drastically and makes a quite novel linkage between spontaneous violation of flavor and supersymmetry breaking. Since the traceless parts of the fermionic components of S are *massless* at tree level (the trace part is eaten by the massive gravitino) they get masses only via radiative corrections and tend to remain very light. They could thus provide very light dark matter candidates as well as cause cosmological problems of the sort typically associated with moduli fields[45]. This requires further detailed study but it is interesting that such a simple extension of the NMSGUT can both reduce free parameters and provide moduli dynamics by taking on the meta-problem of the origin of flavour hierarchy. The structure used entails yet further stringent constraints since the masslessness of the moduli multiplets \hat{S}_{AB} before supersymmetry breaking implies the existence of $N_g(N_g + 1)/2 - 1$ SM singlet fermions generically lighter than the gravitino mass scale and possibly as light as a few GeV. In addition the Polonyi mode S_s may also lead to difficulties in the cosmological scenario. Thus such modes can be both a boon and a curse for familion GUT models. A boon because generic Susy GUT models are hard put if asked to provide Susy WIMPs of mass below 100 GeV as CDM candidates as suggested by the DAMA/LIBRA experiment[35]. A curse because there are strong constraints on the existence of such light moduli which normally demand that their mass be rather large (> 10 TeV) due to the robust cosmological(‘Polonyi’) problems arising from decoupled modes with Planck scale VEVs[36]. In contrast to the simple Polonyi model and String moduli, the BM moduli have explicit couplings to light fields through family D-term mixing and loops. Moreover the MSGUT scenario favours[13, 32] large gravitino masses $> 5 - 50$ TeV. Thus the Polonyi and moduli problems may be evaded. In any case the cosmology need be considered seriously only after we have shown that the MSSM fermion spectrum is indeed generated by the ‘Yukawonified’ NMSGUT[46].

VI. DISCUSSION

The vast range of applicability of the NMSGUT well fits it to claim that it is a theory for all epochs. Not only does it present a well controlled framework for realizing the long standing dream of Grand Unification in a completely realistic fashion, it also makes distinctive predictions concerning the observation of supersymmetry and lepton flavour violation and dynamically palliates the proton decay problem that has chronically afflicted

Susy GUTs without introducing extraneous structures but in a way intrinsic to the GUT itself. Furthermore it provides a surprisingly simple context for the embedding of high scale supersymmetric inflation. The right handed neutrinos and adequate- CKM-PMNS linked- CP violation that are present can enable Leptogenesis. Reaching still further it even serves as a basis for the generation of the flavour hierarchy by the spontaneous violation of a gauged flavour symmetry at the GUT scale and in the process makes the startling suggestion that supersymmetry breaking is linked to flavour violation via a novel BM hidden sector. This implies a striking and novel(for Susy GUTs) prediction of possibly very light non-neutralino Dark Matter candidates. Interestingly the self coupling of this light DM is not constrained. The working out of the various flavour breaking scenarios while maintaining realism in the fermion and GUT SSB sector and accounting for low and high scale threshold corrections is indeed a formidable but manageable project that has already cleared several hurdles[13, 24, 25, 32] that might well have falsified this model in the three decades since its proposal. These successes motivate our belief that a focus on the implication of the consistency conditions that define our world as we know it in the context of the flavour symmetric NMSGUT may further extend the scope of the NMSGUT to include an dynamical understanding of the flavour hierarchy. Moreover the strong reduction in the number of parameters makes falsification much more practicable. Although it continues to live dangerously, we hope that this will be seen as a scientific virtue of the NMSGUT and its generalizations rather than as an unnecessary concreteness and optimism: with which it is sometimes reproached.

-
- [1] Proceedings of UNICOS2014 : Workshop on Unification and Cosmology after Higgs Discovery and BICEP2, May 13-15, 2014, Physics Department, Panjab University, Chandigarh, India.
 - [2] J. C. Pati and A. Salam, Phys. Rev. D **10** (1974) 275 [Erratum-ibid. D **11** (1975) 703].
 - [3] C. S. Aulakh, “Fermion mass hierarchy in the Nu MSGUT. I. The Real core,” hep-ph/0602132.
 - [4] P. Minkowski, Phys. Lett. **B67**,110(1977); M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D.Z. Freedman (North Holland 1979); T. Yanagida, in Proceedings of *Workshop on Unified Theory and Baryon number in the Universe*, eds. O. Sawada and A. Sugamoto (KEK 1979); R.N. Mohapatra and G. Senjanović, Phys. Rev.

- Lett. **44**, 912 (1980); R.N. Mohapatra and G. Senjanović, Phys. Rev. **D23**,165 (1981); G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. **B181**, 287 (1981).
- [5] C. S. Aulakh, A. Melfo and G. Senjanovic, Phys. Rev. D **57** (1998) 4174, [hep-ph/9707256]; C. S. Aulakh, K. Benakli and G. Senjanovic, Phys. Rev. Lett. **79** (1997) 2188 [hep-ph/9703434].
- [6] C. S. Aulakh, B. Bajc, A. Melfo, A. Rasin and G. Senjanovic, Nucl. Phys. B **597** (2001) 89 [hep-ph/0004031].
- [7] C. S. Aulakh, Pramana **54**, 639 (2000) [hep-ph/9903309].
- [8] C. S. Aulakh, Pramana **55**, 137 (2000) [hep-ph/0008331].
- [9] C. S. Aulakh and R. N. Mohapatra, CCNY-HEP-82-4 April 1982, CCNY-HEP-82-4-REV, Phys. Rev. D **28** (1983) 217.
- [10] T.E. Clark, T.K. Kuo, and N. Nakagawa, Phys. Lett. B **115**, 26(1982).
- [11] S. Dimopoulos and H. Georgi, Nucl. Phys. B **193**, 150 (1981).
- [12] C.S. Aulakh, B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, Phys. Lett. B **588**, 196 (2004) [arXiv:hep-ph/0306242].
- [13] C.S. Aulakh and S.K. Garg, [arXiv:hep-ph/0807.0917v2]; Nucl. Phys. B **857**, 101 (2012) [arXiv:hep-ph/0807.0917v3].
- [14] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70** (1993) 2845 [hep-ph/9209215]. This paper initiated serious study of fermion data fitting on the basis of the generic SO(10) formulas for Yukawa couplings. Reasonable fits with just **10** and $\overline{\mathbf{126}}$ Higgs and using Type I neutrino masses were finally found by K.Y. Oda, E. Takasugi, M. Tanaka, M. Yoshimura, Phys. Rev. D 59 (1999) 055001, arXiv:hep-ph/9808241; K. Matsuda, Y. Koide, T. Fukuyama, H. Nishiura, Phys. Rev. D 65 (2002) 033008, arXiv:hep-ph/0108202, Phys. Rev. D 65 (2002) 079904(Errotum); The two generation analysis of the link between third generation charged fermion masses and 2-3 sector neutrino mixing in a $\mathbf{10} \oplus \overline{\mathbf{126}}$ model by B. Bajc, G. Senjanovic, F. Vissani, Phys. Rev. Lett. 90 (2003) 051802, arXiv:hep-ph/0210207 proved extremely influential and was followed by an explosion of interest in the subject. Most notably the work of H.S. Goh, R.N. Mohapatra, S.P. Ng, Phys. Lett. B 570 (2003) 215, arXiv:hep-ph/0303055 and Phys. Rev. D 68 (2003) 115008, arXiv:hep-ph/0308197 and that of K.S. Babu, C. Macesanu, Phys. Rev. D 72 (2005) 115003, arXiv:hep-ph/0505200 resulted in Type I and Type II (as well as mixed) generic tree level three generation fits using only $\mathbf{10} \oplus \overline{\mathbf{126}}$. These fits pre-

dicted that the 1-3 sector mixing angles in the leptonic mixing matrix were most likely large (as was found experimentally in 2012). These original papers provided a strong impetus but in our present understanding quantum threshold effects at M_S, M_X result in mass formulae that obey essentially none of the constraints that follow from the tree level generic analysis. The freedoms assumed by these generic analyses find no support in the explicit computations possible in the (N)MSGUT[13, 25] which show that some of the coefficients assumed in the generic fits are simply unreachable and the MSGUT cannot fit the fermion data [24, 25]. These inabilities were the motivation for the reassignment of roles for the different SO(10) Higgs in the NMSGUT with $\mathbf{10} \oplus \mathbf{120}$ and large $\tan\beta$ driven quantum corrections allowing charged fermion fits and tiny $\overline{\mathbf{126}}$ couplings boosting the Type I neutrino masses to acceptable levels by lowering the right handed neutrino masses below 10^{13} GeV.

- [15] C.S. Aulakh and S.K. Garg, “NMSGUT II: Pinning the Nmsgut@LHC,” [arXiv:hep-ph/0807.0917v1].
- [16] S. Weinberg, Phys. Rev. D **26** (1982) 287. The “dimension 5 ” B-L violating operators noted by Weinberg have generally been accepted to be dangerous since the associated rates are suppressed by only two powers of the superheavy scale rather than the M_X^{-4} suppression characteristic of the classic gauge exchange mediated proton decay. On the other hand there is additional suppression associated with contraction structure and dressing by sparticle exchange required to get the actual dimension 6 4-fermion effective operators responsible for proton decay. Thus the claim of H. Murayama and A. Pierce, that “Not even decoupling can save minimal supersymmetric SU(5) ” (Phys. Rev. D **65** (2002) 055009 [hep-ph/0108104]) has been questioned in B. Bajc, P. Fileviez Perez and G. Senjanovic, Phys. Rev. D **66**, 075005 (2002) hep-ph/0204311 and hep-ph/0210374. They argue that a combination of raising lightest Higgs triplet mass using non-renormalizable terms in the GUT superpotential and/or utilizing the enormous freedom in the soft supersymmetry breaking parameter space can evade the prima facie stringent constraints due to $d = 5$ proton decay in minimal supersymmetric SU(5) GUTs. While this may be strictly true for the minimal(and non realistic) SU(5) model it remains a fact that these operators are much less suppressed by the GUT scale and that if the soft terms are also generated consistently from GUT compatible susy breaking operators then they generically the proton decay rate is 6-8 orders of magnitude larger than current limits. We therefore take the view that a resolution of this issue in terms of the structural

properties of realistic GUTs and a precise study of the quantum corrected relation between GUT and effective MSSM yukawas is an advance in our understanding of GUTs rather than gilding a lily already well enamelled: as maintained by these authors.

- [17] C. S. Aulakh and I. Garg, Phys. Rev. D. **86**, 065001 (2012) [arXiv:hep-ph/1201.0519v4].
- [18] C. S. Aulakh and C. K. Khosa, Phys. Rev. D **90**, 045008 (2014) [arXiv:1308.5665 [hep-ph]].
- [19] J. C. Bueno Sanchez, K. Dimopoulos and D. H. Lyth, JCAP **0701**, 015 (2007) [hep-ph/0608299].
- [20] C. S. Aulakh, Phys. Rev. D **91** (2015) 5, 055012 [arXiv:1402.3979 [hep-ph]].
- [21] C. S. Aulakh, “Truly minimal unification: Asymptotically strong panacea?,” hep-ph/0207150.
- [22] C.S. Aulakh and A. Girdhar, [arXiv:hep-ph/0204097]; v2 August 2003; v4, 9 February, 2004; Int. J. Mod. Phys. A **20**, 865 (2005).
- [23] C.S. Aulakh and A. Girdhar, Nucl. Phys. B **711**, 275 (2005) [arXiv:hep-ph/0405074].
- [24] C. S. Aulakh, hep-ph/0506291.
- [25] C. S. Aulakh and S. K. Garg, Nucl. Phys. B **757** (2006) 47 [hep-ph/0512224].
- [26] B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, Phys. Rev. D **70**, 035007 (2004) [arXiv:hep-ph/0402122].
- [27] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, Eur. Phys. J. C **42**, 191 (2005) [arXiv:hep-ph/0401213v1,v2]; T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, J. Math. Phys. **46** (2005) 033505 [arXiv:hep-ph/0405300].
- [28] Charanjit Kaur Khosa, PhD Thesis, Panjab University, defended February 2015, ArXiv 1506.04101.
- [29] Ila Garg, PhD Thesis, Panjab University, defended March 2015, ArXiv 1506.05204.
- [30] B.D. Wright, [arXiv:hep-ph/9404217] (1994).
- [31] C.S. Aulakh, “*NMSGUT-III: Grand Unification upended*,” [arXiv:hep-ph/1107.2963].
- [32] C. S. Aulakh, “NMSGUT III: Grand Unification Upended” [arXiv:hep-ph/1107.2963v1].
C. S. Aulakh, I. Garg and C. K. Khosa, Nucl. Phys. B **882**, 397 (2014) [arXiv:hep-ph/1311.6100].
- [33] S. Ray, “Some properties of meta-stable supersymmetry-breaking vacua in Wess-Zumino Phys. Lett. B **642** (2006) 137 [arXiv:hep-th/0607172].
- [34] B. Bajc and A. Melfo, “Metastable gauged O’Raifeartaigh,” JHEP **0804**, 062 (2008) [arXiv:0801.4349 [hep-ph]].

- [35] R. Bernabei, P. Belli, S. d’Angelo, A. Di Marco, F. Montecchia, F. Cappella, A. d’Angelo and A. Incicchitti *et al.*, “Dark Matter investigation by DAMA at Gran Sasso,” *Int. J. Mod. Phys. A* **28**, 1330022 (2013) [arXiv:1306.1411 [astro-ph.GA]].
- [36] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, “Cosmological Problems for the Polonyi Potential,” *Phys. Lett. B* **131** (1983) 59; B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, “Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings,” *Phys. Lett. B* **318** (1993) 447 [hep-ph/9308325].
- [37] C. S. Aulakh, I. Garg and C. K. Khosa(to appear).
- [38] C. S. Aulakh, I. Garg and C. K. Khosa(to appear).
- [39] J. Martin, C. Ringeval and V. Vennin, *Phys. Dark Univ.* (2014) [arXiv:1303.3787 [astro-ph.CO]].
- [40] PLANCK Collaboration [arXiv:1405.0871v1]; [arXiv:1405.0872v1]; [arXiv:1405.0873v1]; [arXiv:1405.0874v2].
- [41] R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, *Phys. Rev. Lett.* **97** 191304 (2006) [arXiv:hep-ph/0605035]; R. Allahverdi, B. Dutta, A. Mazumdar, *Phys. Rev. D* **75**, 075018 (2007). [hep-ph/0702112 [HEP-PH]].
- [42] R. Allahverdi, A. Kusenko and A. Mazumdar, *JCAP* **0707** 018 (2007) [arXiv:hep-ph/0608138].
- [43] P. A. R. Ade *et al.* [BICEP2 Collaboration], *Phys. Rev. Lett.* **112**, 241101 (2014) [arXiv:1403.3985 [astro-ph.CO]].
- [44] Y. Koide, *Phys. Rev. D* **78**, 093006 (2008) [arXiv:0809.2449 [hep-ph]]; *Phys. Rev. D* **79**, 033009 (2009) [arXiv:0811.3470 [hep-ph]]; *Phys. Lett. B* **665**, 227 (2008).
- [45] G.D. Coughlan, W. Fischler, E.W. Kolb, S. Raby and G. G. Ross, *Phys. Lett. B* **131** (1983) 59; B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, *Phys. Lett. B* **318** (1993) 447 [hep-ph/9308325].
- [46] C. S. Aulakh and C. K. Khosa, “SO(10) grand unified theories with dynamical Yukawa couplings,” *Phys. Rev. D* **90**, no. 4, 045008 (2014) [arXiv:1308.5665 [hep-ph]].